

Latent predictors: measurement error plus identification

$$\tilde{X} = X + U$$

$$\hat{X} = \frac{\tilde{X} - \overline{\tilde{X}}}{\text{sd}(\tilde{X})}$$

$$\text{Var}(X) = 1 \quad \text{and} \quad \text{Var}(\hat{X}) = 1$$

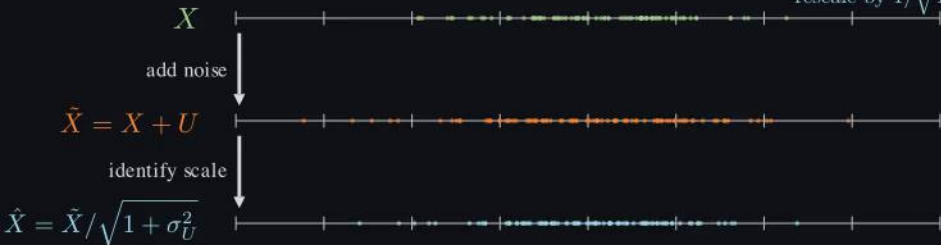
The noisy estimate spreads out, then identification rescales it back to unit variance.

1. Noisy latent estimates are re-scaled for identification

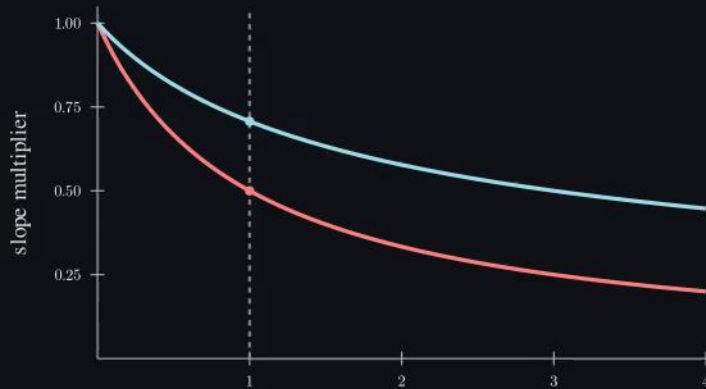
$$\sigma_U^2 = 0.55$$

$$\text{sd}(\tilde{X}) = \sqrt{1 + \sigma_U^2} = 1.24$$

$$\text{rescale by } 1/\sqrt{1 + \sigma_U^2} = 0.80$$



2. Identification changes the attenuation factor



$$\text{classical : } (1 + \sigma_U^2)^{-1}$$

$$\text{latent : } (1 + \sigma_U^2)^{-1/2}$$

With identified latent scores, variance only the covariance loss remains.

$$\sigma_U^2 = 1.00$$

$$\lambda_{\text{classical}} = 0.50$$

$$\lambda_{\text{latent}} = 0.71$$

3. Split indicators estimate the correction factor

$$\tilde{X}_1 = X + U_1, \quad \tilde{X}_2 = X + U_2$$

$$\rho = \text{Cor}(\hat{X}_1, \hat{X}_2) = \frac{1}{1 + \sigma_U^2}$$

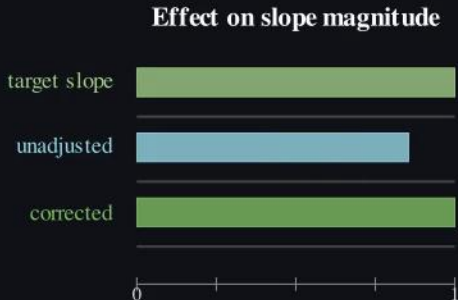
$$\text{plim } \hat{\beta}_{\hat{X}} = \sqrt{\rho} \beta_X$$

$$\hat{\beta}^* = \frac{\hat{\beta}_{\hat{X}}}{\sqrt{\rho}}$$

$$\rho = 0.73$$

$$\sqrt{\rho} = 0.85$$

$$\hat{\beta}^* / \beta_X = 1.00$$



As measurement error grows, the unadjusted slope shrinks.

The split-correlation correction restores the latent-scale slope.

Takeaway

1. Latent estimates are noisy measurements of an identified trait.
2. Standardization changes the measurement-error attenuation factor.
3. Split indicators estimate reliability, so slopes can be corrected by $1/\text{sqrt}(\rho)$.

$$\rho = \text{Cor}(\hat{X}_1, \hat{X}_2), \quad \hat{\beta}^* = \frac{\hat{\beta}_{\hat{X}}}{\sqrt{\rho}}$$