Measurement Error in Latent Predictors: The Role of Identification Restrictions

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Latent Traits in Political Science

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* Latent traits typically estimated using multiple indicators
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* Latent variables have no objective scale
  - Investigator must use identifying restrictions
    ⇝ Normalization constraints (mean 0, sd 1)

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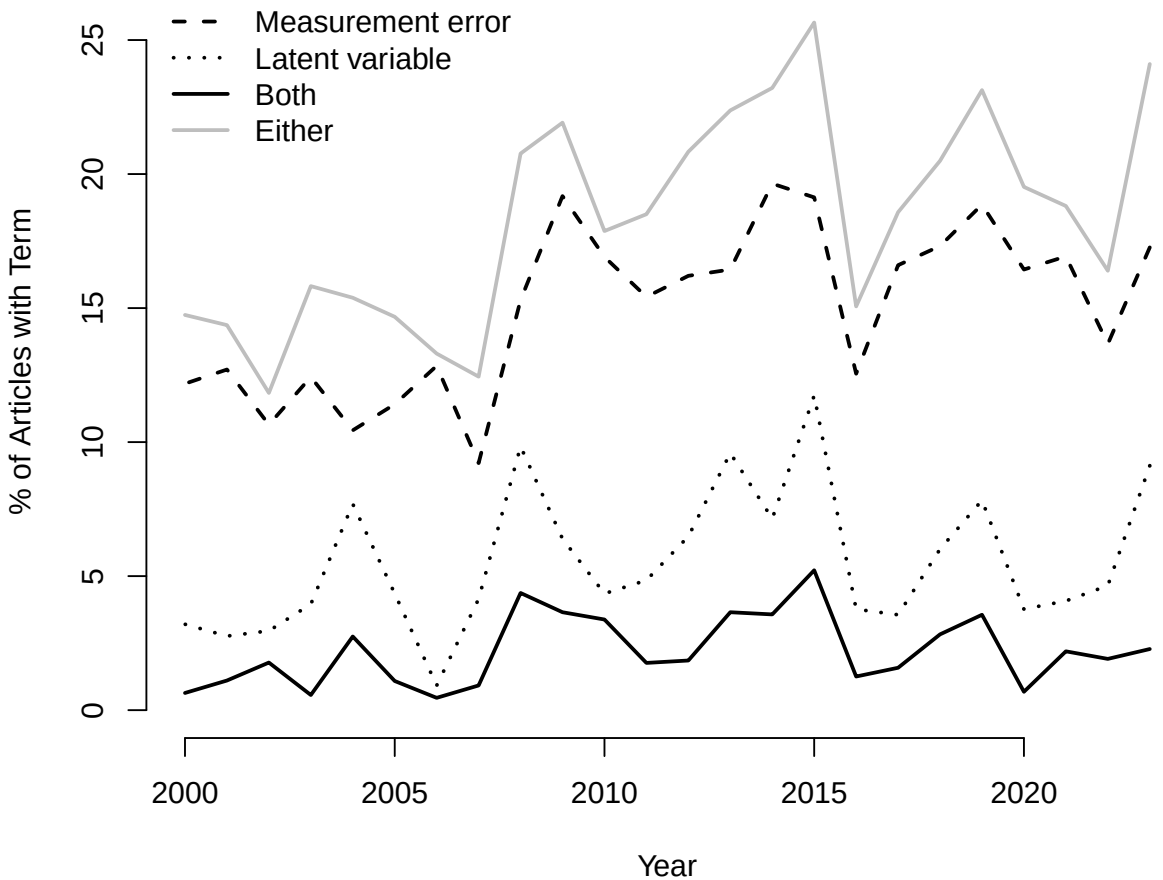

- TAKEAWAY: Growing interest in measurement error; less work on measurement error in latent estimates

Measurement Error in Regression Models: Standard Case

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  - where $U \perp\!\!\!\perp X$, $\mathbb{E}[U] = 0$ and $\mathbb{V}(U) = \sigma_U^2$
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* Regressing Y on $\tilde{X}$ (not true X),

  Standard Attenuation Bias: $\mathbb{E}[\hat{\beta}_{\tilde{X}}] = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_U^2}\ \beta_X$

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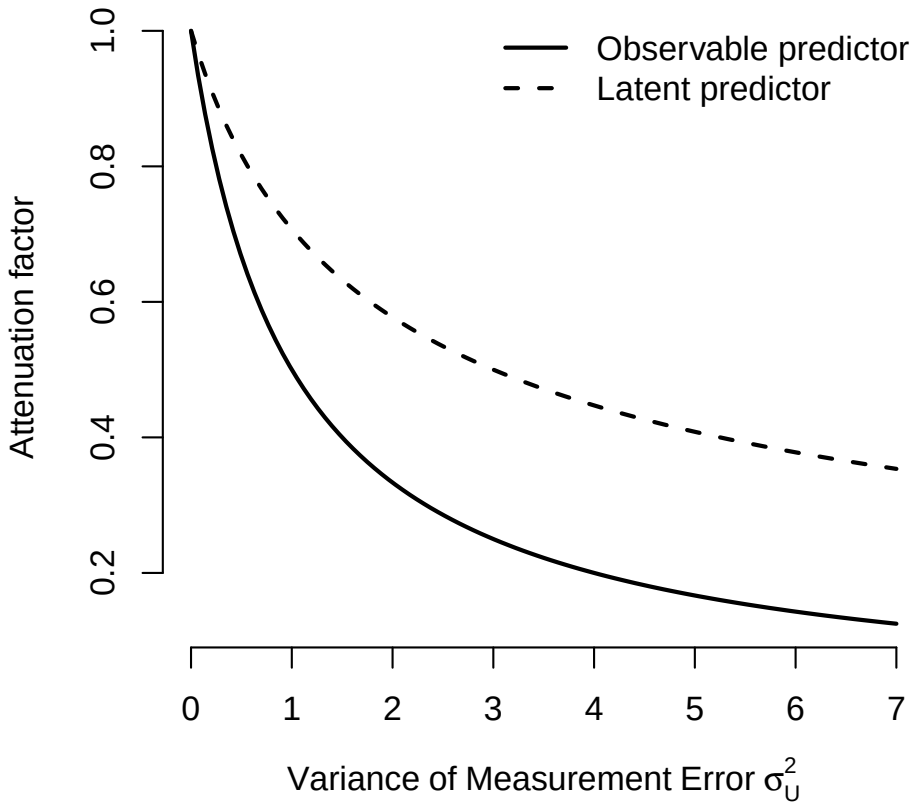
* Therefore: Attenuation with Latent Variables:

$$\mathbb{E}[\beta_{\hat{X}}] = \mathbb{E}[\beta_{\tilde{X}}] \sqrt{1 + \sigma_U^2} = \frac{1}{1 + \sigma_U^2} \beta_X \sqrt{1 + \sigma_U^2}$$

$$= \frac{1}{\sqrt{1 + \sigma_U^2}} \beta_X$$

Attentuation Factor with Latent Space Predictor



- ATTENUATION FACTOR: $\widehat{\beta}/\beta$ (1 is unbiased)

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  - IV Recipe: Split indicators into 2 sets
    - Estimate X separately based on each set, producing $\hat{X}_1, \hat{X}_2$
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    - Define $X \equiv \hat{X}_1, Z \equiv \hat{X}_2$
* **But** in Latent Case with $\widehat{X}$ IV *over*-corrects $\rightsquigarrow$ inflation bias

  $\hat{\beta}_{\mathrm{IV},\hat{X}} = \hat{\beta}_{\mathrm{IV},\tilde{X}} \sqrt{1 + \sigma_U^2}$;

  Hence $\operatorname{plim} \hat{\beta}_{\mathrm{IV},\hat{X}} = \beta_X \sqrt{1 + \sigma_U^2}$,

  where $\sqrt{1 + \sigma_U^2} > 0$ whenever $\sigma_U^2 > 0$

Illustrative Simulation Results: Ideology & Vote

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* Context: Based on 2016 CCES:
  1,000 Respondents, 30 Indicators
  - $Y_i =$ Presidential Vote (1=Trump, 0=Clinton)
  - $X_i =$ Ideology (mean 0, var 1, higher = conservative)
* Goal: *Estimate slope coefficient in regression:*

$$Y_i = \beta_0 + X_i \beta_X + \epsilon_i$$

* Indicators: P, generated from ideal point model:

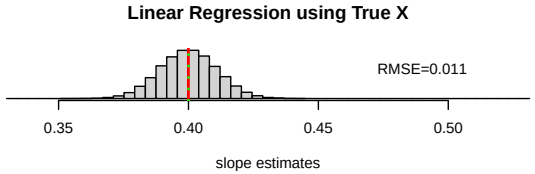
$$\Pr(P_{id} = 1) = \Phi\left(X_i \eta_d + \alpha_d\right)$$

Set true values of all parameters based on estimates from the actual data (100k Monte Carlo sims)

Illustrative Simulation Results: Ideology & Vote

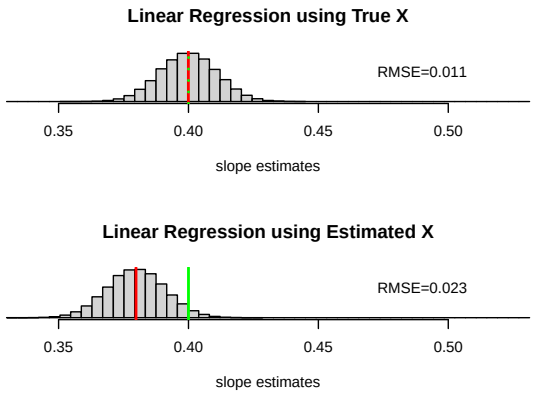
* TRUE X
  assume we knew true values for ideology (X)
  - Unbiased estimation of β_X with high precision (low RMSE)

  ...of course, we don't ever really observe the true X, but have to estimate it



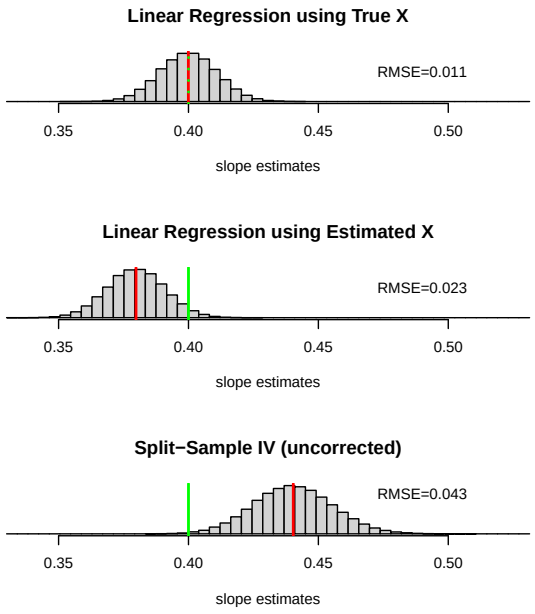
Illustrative Simulation Results: Ideology & Vote

* "Naive" Estimator
  use $\hat{X}$ as our predictor instead of the true X?
  - Attenuation bias – estimates too close to zero on average
  - RMSE more than double what it was when we knew true X



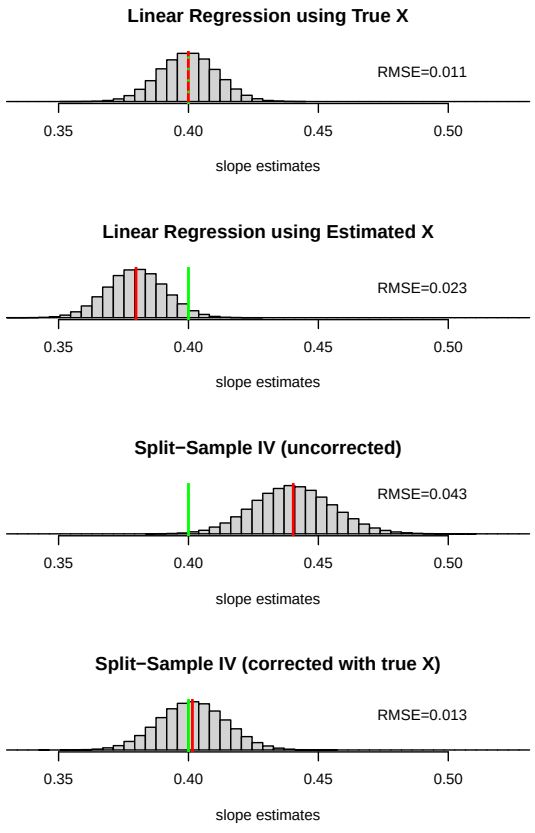
Illustrative Simulation Results: Ideology & Vote

* Split-Half IV, Uncorrected
  - Estimate $\hat{X}_1, \hat{X}_2$ each based on half of indicators
  - Define $X \equiv \hat{X}_1, Z \equiv \hat{X}_2$
- *Over*-adjusts
  - Ignores effects of identification on attenuation
  - RMSE nearly 4$\times$True X estimator, double the naive estimator



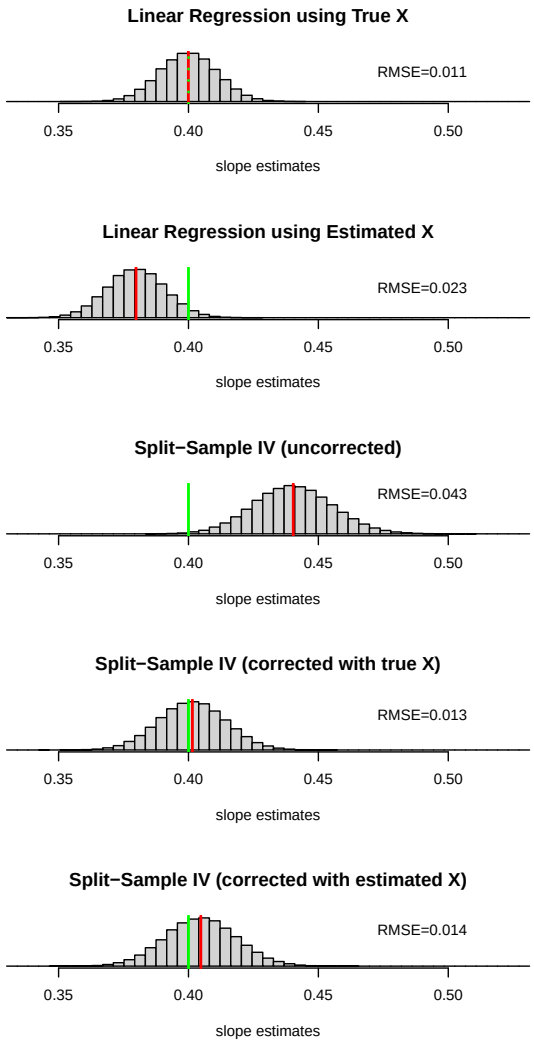
Illustrative Simulation Results: Ideology & Vote

* Split-Half IV, **Corrected** using true X
  - Same split-half IV setup
  - Now $\rightsquigarrow$ Estimate σ_U^2 with variance of $\hat{X}_1 - X$
  - Cheating – uses unknown X!
- *Very low bias, high precision*
  - RMSE barely higher than True X case



Illustrative Simulation Results: Ideology & Vote

* Split-Half IV, **Corrected** Using $\hat{X}_1, \hat{X}_2$
  - Same split-half IV setup
  - Now, estimate σ_U^2 with $\mathbb{V}(\hat{X}_1 - \hat{X}_2)/2$
  - This is **not** cheating
  - ...but does assume measurement error in $\hat{X}_1, \hat{X}_2$ is $\perp$ and has same var.
- *Very low bias, quite precise*
  - RMSE barely higher than True X case



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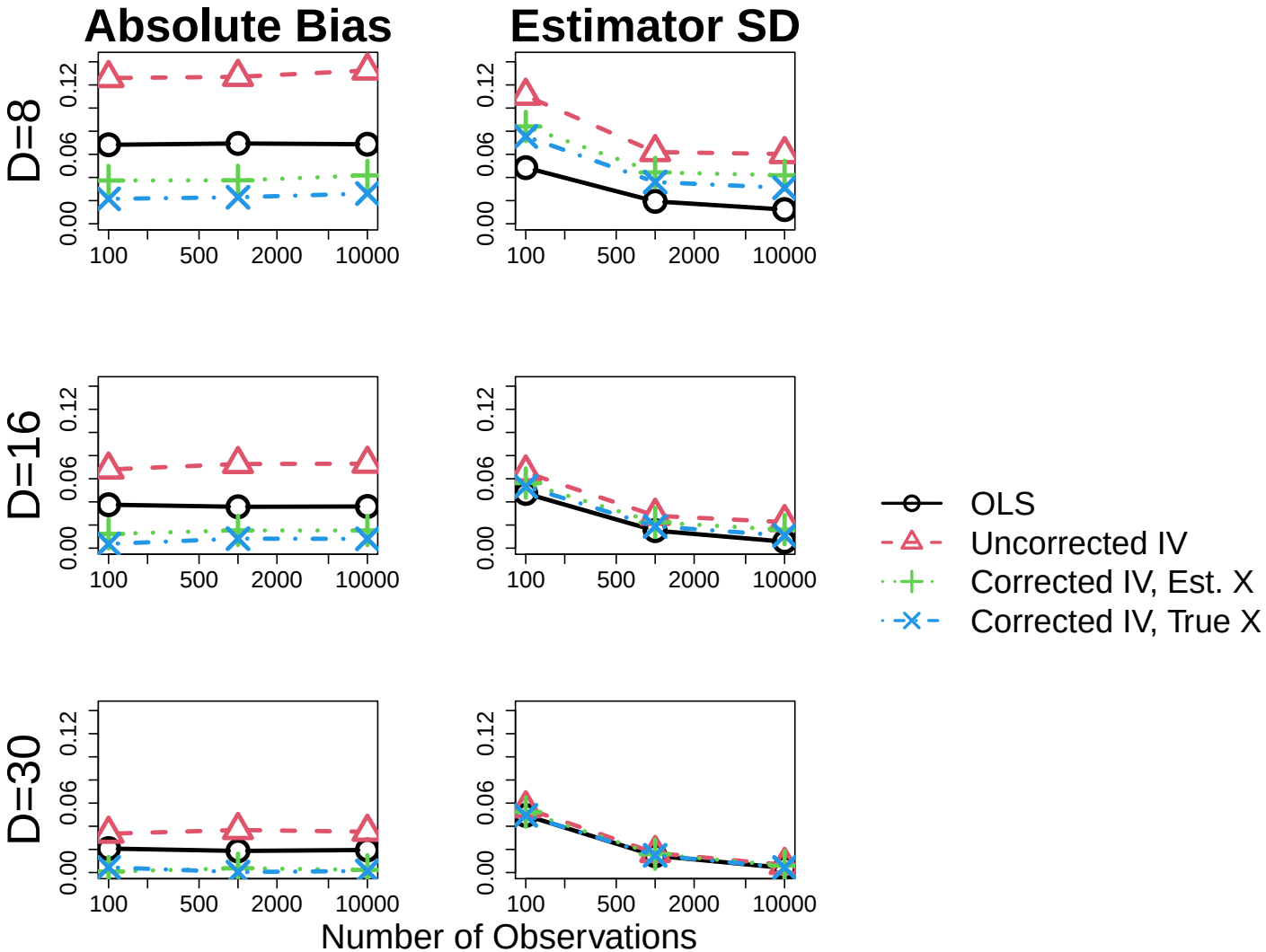

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Table: Percentage change in effect estimate of ideology on vote choice, corrected IV vs. OLS. Rows correspond to observation number; columns to dimensions used in estimating latent ideology.

	13	21	30
60	0.323	0.188	0.132
519	0.29	0.163	0.103
4491	0.296	0.163	0.101
38851	0.302	0.166	0.104

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519	-0.154	-0.122	-0.099
4491	-0.155	-0.123	-0.101
38851	-0.154	-0.124	-0.1

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For more information:

laits.utexas.edu/sjessee & ConnorJerzak.com

Appendix Slides

Democracy Analysis: Corrected IV vs. OLS

Table: Percentage change in effect estimate of democracy on growth (long difference), corrected IV vs. OLS. Rows correspond to observation number; columns to dimensions used in estimating latent democratic status.

	4	8	13	17
60	0.093	0.032	0.013	0.011
91	0.072	0.032	0.013	0.009
137	0.098	0.032	0.01	0.012
208	0.073	0.028	0.012	0.01

Democracy Analysis: Corrected vs. Uncorrected IV

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	4	8	13	17
60	-0.092	-0.047	-0.029	-0.022
91	-0.091	-0.044	-0.028	-0.022
137	-0.1	-0.044	-0.028	-0.021
208	-0.087	-0.043	-0.027	-0.021